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ADAPTIVE NONLINEAR OPTIMAL CONTROL FOR A BANANA DEHYDRATION PROCESS

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Abstract. *The contribution presents the adaptive scheme of the optimal nonlinear control synthesized via inverse optimality approach to regulate the temperature of a banana dehydration process. The necessity to apply an adaptive approach is evident when the desire operation region is frequently reset, due to the system parameter changes, requiring under this condition a new tuning of the controller gains. Then, the proposed adaptive algorithm provides a greater ease to be implemented, estimating the system parameters on line, and also has more robustness in contrast to an adaptive linear optimal strategy. The obtained experimental results provide a convincing evidence of the feasibility of the adaptive design.*
Keywords: Banana dehydration process, Energy savings, Adaptive nonlinear and linear optimal control

1. **Introduction.** Drying operation is an energy intensive process, representing from 10% to 25% of energy consumption [1] in industrial processes of developed countries. Hence, for optimal drying conditions, an optimal controller is preferred to decrease drying time and fuel consumption [2]. Recently, some optimal [3, 4] and suboptimal [5] controllers, regulating the air temperature of a sliced tomatoes dehydrator, have been reported. However, the optimal controllers presented in [3, 4] were synthesized via inverse optimality approach, in which a complete and a reduced type control Lyapunov Krasovskii functionals have been constructed and proposed, respectively. In the case of the complete type functional reported in [3], it is necessary to find three positive definite matrices, positive scalars for the Single Input-Single Output (SISO) systems, and this fact makes difficult their implementation. Another problem is the dependence of the control design from the plant model; the reason is that for a specific test, when the Set Point (SP) has changed, a new mathematical model must be identified for the new operation zone. In [4] the last fact was improved; nevertheless, the proposing of matrices, which define the

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Control Lyapunov-Krasovskii Functional (CLKF), is not an easy task in order to guarantee the feasibility of the Linear Matrix Inequality (LMI) problem which is a sufficient condition to conclude that the Lyapunov-Krasovskii functional is a CLKF. Besides, in the best knowledge of authors, the CLKF and optimal control approaches have not been yet applied on the banana dehydration process. In fact, by the inverse optimality and control Lyapunov-Krasovskii functionals approaches, in [3, 4] was exposed how to synthesize the optimal controls which regulate the drying air temperature of the tomatoes dehydration process; experimental results give evidence of important energy savings and good performance when the optimal controls performances were compared to industrial controllers. In [6] the use of a predictive optimal control to regulate the drying air temperature in a tomatoes dehydration process was proposed, and experimental results were presented. With respect to bananas, in [7] the influence of air-drying of banana slices of experimental parameters such as temperature, relative humidity and slices thickness was presented; however, the use of optimal control on banana dehydration process is incipient yet. The last point is our main motivation to propose the using of optimal control in a banana dehydration process, but, as it is exposed above, the conditions which have to be satisfied in order to implement the nonlinear optimal control in this type of process, could be discouraging. The reason for this, is that when the user changes the SP, a new process model identification has to be do; furthermore, new parameters related with the controller design have been proposed. So, in this article to overcome these difficulties, the using of classical identification AutoRegressive eXogenous (ARX) model is proposed. In fact, the optimal nonlinear control proposed in [3] is used here, but an online identification mathematical model of the plant is used. Additionally, the sufficient condition to conclude that the complete type Lyapunov-Krasovskii functional is CLKF is verified online, and all the scheme is implemented in a Programmable Automatic Control (PAC) MyRIO from National Instruments with Laboratory Virtual Instrument Engineering Workbench (LabVIEW) environment. Experimental results allow to conclude the feasibility of our proposal. Moreover, a comparison with an optimal adaptive linear control is made and some advantages are concluded.

Notation: C is the space of continous functions, $PC\left([-h,0],\mathbb{R}\right)$ is the space of piecewise continuous functions defined on the segment $[-h,0]$, the restriction of $x(t)$ is $x_t = x(t+\theta)$, $\theta \in [-h,0]$, the standard uniform norm is $\|\varphi\|_h = \sup\limits_{\theta\in[-h,0]} \|\varphi(\theta)\|$.

The paper is organized as follows. Section 2 describes the banana dehydration process and its mathematical model. Section 3 presents the optimal nonlinear control and its adaptive version is presented in Section 4. The implementation details and the experimental results obtained in a dehydrator process are illustrated in Section 5, and finally some concluding comments are presented in Section 6.

2. **Banana Dehydration Process.** The banana atmospheric dehydration process consists to pass hot air through a product until the humidity content is reduced to 15% [14]. The dehydrator platform is made of stainless steel, and it consists of a drying chamber with the following dimensions: a box of 40 cm × 35 cm × 40 cm containing the product, and a heating chamber (40 cm × 35 cm × 40 cm) where the electrical grid is located. The distance between the electrical grid and the product is 25 cm; a wind tunnel as output (diameter of 10 cm), and a pipe that recycles the hot air into the system (diameter of 10 cm and length of 80 cm), this pipe optimizes the energy using by the system, but it induces a state delay which depends on its length. The dehydration system is also composed by a temperature sensor $LM35$ with a measurement rate of 10 mV/°C; a fan producing a constant air flow of 2.1 m/s; an actuator, which is an electrical grid used as a

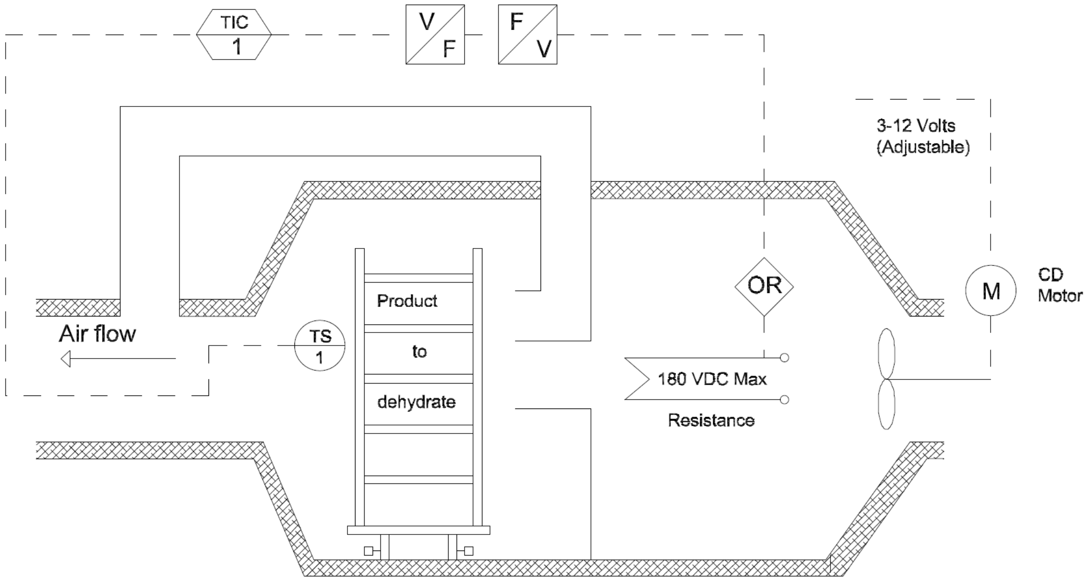

FIGURE 1. Instrumentation diagram of the dehydration prototype

heat source; a PAC MyRIO, a real-time embedded board made by National Instruments; LabVIEW software made by National Instruments for the implementation of the control law and manages the data storage; all these hardwares are used to implement the optimal controllers. The temperature inside the chamber is regulated by the applied voltage to the electrical grid, which is in the range of 0-180 volts of direct current (control signal). The instrumentation diagram of the dehydration system is shown in Figure 1. Broadly, the process operation description is described as follows: an electric grid (the control signal is applied on it) heats the air produced by the fan; the heated air is sent to the plate where the banana slices are located, and where the air temperature is sensed; now, one part of the air leaves through the wind tunnel and the other part returns to the process by the recycling pipe. The temperature level is measured every 500 milliseconds and it is used for the control algorithms to compute the voltage level to be applied to the actuator.

In this prototype the two control algorithms are tested: the adaptive optimal nonlinear and the adaptive optimal linear controllers. As both strategies require the mathematical model of the plant, the nonlinear mathematical model proposed in [5] is used and it is briefly recalled here. Considering the first law of thermodynamics, the energy conservation principle [8], and some assumptions (see details in [5]) about the plant, following model can be obtained:

$$\frac{dT(t)}{dt} = \bar{f}_0(T(t)) + \bar{f}_1(T(t-h)) + \bar{b}(T(t))u(t) + \bar{f}_2(T(t),t), \tag{1}$$

where the voltage level $u(t)$ applied to the electrical grid is assumed to satisfy the linear relation $u(t) = \hat{k}\bar{Q}(t)$. Furthermore, the heating rate is defined by $\bar{Q}(t)$ and $\hat{k}$ is a proportionality constant, $\bar{b}(T(t)) = \left(\frac{1}{V\rho\frac{d}{dT}\left(\frac{\partial H}{\partial T}\right)}\right)$, $\bar{f}_1(T(t-h)) = \rho_0\bar{F}_0(t-h)\frac{\partial H_0(t)}{\partial T}\bar{b}(T(t))$, and

$$\bar{f}_0(T(t)) = \left(\left(\bar{c}\rho_0\frac{\sqrt{(P_a(t) - P_c(t))}}{\sqrt{\rho_a}} - \rho F(t)\right)\frac{\partial H_0(t)}{\partial T}\right)\bar{b}(T(t)),$$

where $T(t)$ is the temperature of the drying air, $\bar{c} = \frac{\sqrt{2}CE\varepsilon\pi d^2}{4}$, C, E, ε are the discharge coefficients, speed of approach, and expansion of air during the acceleration of the flow, respectively; P_a and P_c are the pressures before and after the nozzle respectively (the nozzle discharged the hot air to the product), ρ_a is the density of the fluid before the nozzle, $H(t)$ is the enthalpy related with the air flow in the heater chamber (where the electric grid is), and $H_0(t)$ is the enthalpy related with drying chamber (where the product is), ρ_0 is the air density, V is the volume air, $F(t)$ is the volumetric flow in the heater chamber, and $F_0(t)$ is the volumetric flow in the drying chamber. It is assumed that the feedback flow $\bar{F}_0(t-h)$ induces a delay in the temperature, and $\bar{f}_2(T(t),t) = \bar{b}(T(t))T_e(t)$, where $T_e(t)$ is the external temperature. The difficulty with the model (1) is in the thermodynamic magnitudes, which are not generally found in a direct way, and there are partial derivatives of enthalpy with respect to temperature, whose numeric values are difficult to calculate. Simpler models can be obtained by means of a Taylor series expansion around an equilibrium point and thus obtain models with a linear part and nonlinear terms [3]:

$$\dot{x}(t) = a_0x(t) + a_1x(t-h) + bu(t) + g(x(t), x(t-h)), \tag{2}$$

where $a_0, a_1, b \in \mathbb{R}$ are the parameters to be determined, $h > 0$ is the time delay, the state $x(t)$ is the temperature $T(t)$, the control input $u(t)$ is the voltage applied to the electrical grid, and according to [9], the nonlinear dynamic of $g(\cdot,\cdot)$ can be described by a polynomial function

$$g(x(t), x(t-h)) = \gamma_0x^2(t) + \gamma_1x^2(t-h) + \gamma_2x^3(t) + \gamma_3x^3(t-h) \tag{3}$$

For the tomatoes case [3], the model (2), was identified off-line and the nonlinear optimal controller is synthesized with the identified parameters. However, when the SP changes, the operation temperature T_{op} is different and consequently the mathematical model around this new operating point changes. This change means that the controller gains have to be recalculated. This is a motivation to implement an adaptive control, which requires the online model parameters identification.

3. **Optimal Nonlinear Control.** The linear part of the system given by (2): $\dot{x}(t) = a_0x(t) + a_1x(t-h)$ is assumed to be stable; further, it is associated with a complete type Lyapunov Krasovskii functional with prescribed derivative [11]. In [3] the complete type Lyapunov Krasovskii, Control Lyapunov-Krasoskii Functional (CLKF) and inverse optimality approaches are combined in order to obtain a nonlinear optimal control to the models (2) and (3). Additionally, sufficient conditions were given to conclude that a complete type Lyapunov Krasosvkii functional is a CLKF. Here below, these concepts are briefly recalled.

3.1. **Lyapunov-Krasovskii functionals of complete type.** Consider the open loop linear part of the system (2) defined as follows

$$\dot{x}(t) = a_0x(t) + a_1x(t-h), \quad t \geq 0, \tag{4}$$

where $h > 0$ is the known delay, and a_0, a_1 are real numbers. Let $\varphi : [-h, 0] \to \mathbb{R}$ be an initial function, $\varphi \in \mathcal{PC}([-h, 0], \mathbb{R})$, then $x(\theta) = \varphi(\theta)$, $\theta \in [-h, 0]$. The idea of the complete type functional approach could be summarized as follows: given a stable linear time delay system and a positive definite functional $w(x_t)$, the complete type functional has time derivative along the trajectories of system (4) equal to $-w(x_t)$ and the functional has lower and upper quadratic bounds [11]. Following theorem gives explicitly the form of the complete type functional $V : \mathcal{PC}([-h, 0], \mathbb{R}) \to \mathbb{R}$ for the scalar case.

Theorem 3.1. [11] *Given three real numbers W_j, $j = 0, 1, 2$, let us define the functional*

$$w(\varphi) = \varphi^2(0)W_0 + \varphi^2(-h)W_1 + \int_{-h}^{0} \varphi^2(\theta)d\theta W_2.$$

If there exists a function $U(\tau)$ associated with $W = W_0 + W_1 + hW_2$, the functional

$$V(\varphi) = \varphi^2(0)U(0) + 2a_1\varphi(0)\int_{-h}^{0} U(-h-\theta)\varphi(\theta)d\theta + \int_{-h}^{0} \varphi^2(\theta)\left[W_1 + (h+\theta)W_2\right]d\theta + a_1^2\int_{-h}^{0}\varphi(\theta_1)\left[\int_{-h}^{0} U(\theta_1-\theta_2)\varphi(\theta_2)d\theta_2\right]d\theta_1, \quad (5)$$

has time derivative along the solutions of system (4) given by $\frac{dV(x_t)}{dt} = -w(x_t)$, $t \geq 0$.

Here $U(\tau)$, the Lyapunov matrix of system (4) associated to $W = W_0 + W_1 + hW_2$ is the unique solution of the symmetry, dynamic, and algebraic properties [11], and functional (5) has lower and upper bound

$$\alpha_1 \|\varphi(0)\|^2 \leq V(\varphi) \leq \alpha_2 \|\varphi\|_h^2, \ \varphi \in \mathcal{PC}([-h,0],\mathbb{R}), \ \alpha_1, \alpha_2 > 0.$$

3.2. **Control Lyapunov-Krasovskii functionals.** In order to use the CLKF approach, consider the time delay system (2), it can be rewritten as an affine system in the control input as follows

$$\dot{x}(t) = f_0(x_t) + bu(t), \quad (6)$$

where

$$f_0(x_t) = a_0x(t) + a_1x(t-h) + g(x(t), x(t-h)), \quad (7)$$

Here, the system has zero solution when $u \equiv 0$. The control $u \in \mathbb{R}$ is a piece wise continuous of the state function and the initial condition is given by a continuous vector valued function $x_0 = \varphi$, $\varphi : [-h,0] \to \mathbb{R}$. The idea is to consider a CLKF as a Bellman functional and the dynamic programming approach with a specific performance index to satisfy the Hamilton-Jacobi-Bellman (HJB) type equation without solving it. The time derivative of the functional (5) along the trajectories of system (6) is given by

$$\left.\frac{dV(x_t)}{dt}\right|_{(6-7)} = \Psi_0(x_t) + \Psi_1(x_t)u(t),$$

where

$$\Psi_0(x_t) = -x^2(t)W_0 - x^2(t-h)W_1 - \int_{-h}^{0} x^2(t+\theta)d\theta W_2 + 2\omega_1(x_t)g(x(t), x(t-h)), \quad (8)$$

and

$$\Psi_1(x_t) = 2b\left(U(0)x(t) + a_1\int_{-h}^{0} U(-h-\theta)x(t+\theta)d\theta\right), \quad (9)$$

are both scalar functionals. If the functional $\Psi_1(x_t) \neq 0$ for all $x_t \neq 0$, a nonlinear control law can be applied, when the functional $\Psi_1(x_t) = 0$ but $x_t \neq 0$, the sufficient condition that guarantees that the system (6) is asymptotically stable and consequently to conclude that the complete Lyapunov functional (5) is a CLKF, is that the term $\Psi_0(x_t)$ is negative. This sufficient condition (recalled here for the scalar case) is established in the following proposition.

Proposition 3.1. [3] *Let the nonlinear time delay system (6) and let positive numbers $W_j \in R$, for $j = 0, 1, 2$, and $W = W_0 + W_1 + hW_2$ be given. If there exists a scalar $\epsilon > 0$*

such that the matrix

$$E = \begin{bmatrix} W_0 - \epsilon\bar{\alpha} & 0 & 0 & -U(0) \\ 0 & W_1 - \epsilon\bar{\beta} & 0 & 0 \\ 0 & 0 & \frac{1}{h}\frac{W_2}{\bar{s}^2} & -1 \\ -U(0) & 0 & -1 & \epsilon \end{bmatrix} > 0. \qquad (10)$$

then the complete type functional $V(x_t)$ given by (5) is a control Lyapunov-Krasovskii functional for systems (6) and (7).

Remark 3.1. [3] *The numerical values for W_i, for $i = 0, 1, 2$, and ϵ could be found by considering (10) as an LMI, under the restriction $W = W_0 + W_1 + hW_2$. However, in practice these numerical values could introduce a stable behavior in the plant, but without adequate performance in closed loop. This problem may be solved, by introducing an adaptive scheme in the control loop.*

3.3. **Optimal nonlinear control law via inverse optimality approach.** Consider the following performance index

$$J = \int_0^\infty \left[q(x_t) + r(x_t)u^2\right] dt, \qquad (11)$$

where $q(x_t)$ and $r(x_t)$ are strictly positive definite [3] and these depend on the time derivative of $V(x_t)$; therefore, they are well defined by

$$q(x_t) = \left[\Psi_1(x_t)^2\right] + \sqrt{\left[\Psi_0(x_t)\right]^2 + \Psi_1^4(x_t)},$$

$$r(x_t) = \frac{\frac{1}{4}\left[\Psi_1^2(x_t)\right]}{\Psi_1^2(x_t) + \Psi_0(x_t) + \sqrt{\left[\Psi_0(x_t)\right]^2 + \Psi_1^4(x_t)}},$$

with $\Psi_0(x_t)$ and $\Psi_1(x_t)$ given by Equations (8) and (9) respectively. If Proposition 3.1 is satisfied, then the inverse optimality approach can be used to obtain an optimal control for system (6), which is presented below.

Proposition 3.2. [3] *Suppose that the functional $V(x_t)$ given by (5) satisfies the condition established in Proposition 3.1, then the optimal control law*

$$u^*(t) = \begin{cases} -\frac{1}{2}\frac{\Psi_1(x_t^*)}{r(x_t^*)}, & \Psi_1(x_t^*) \neq 0 \\ 0, & \Psi_1(x_t^*) = 0, \ \text{or } x_t^* = 0 \end{cases}, \qquad (12)$$

stabilizes system (6), in the local sense, and minimizes the performance index (11).

Remark 3.2. [3] *The control law given by (12) is continuous in $x_t^* = 0$. In fact,* [3] *proved that the complete type functional $V(x_t^*)$ satisfies the small control property* [12], *and it follows that u^* is continuous at $x_t^* = 0$.*

4. **Adaptive Optimal Nonlinear Control Scheme.** As it was previously mentioned in Remark 3.1, the numerical values of the control gains W_i, (with $i = 0, 1, 2$), could be found by satisfying the condition (10); however, the obtained values by this way could be inadequate to the plant response performance, producing either saturation in the actuator or small values that not are useful. This is one of reasons to propose an adaptive scheme.

Additionally, when the set point is reset, the operation region changes, together with the parameters of the mathematical model. In order to obtain good results, in [3] a new model was computed for every operation point; however, the numerical values for the positive numbers W_i, (with $i = 0, 1, 2$), must be found for every model to satisfy the sufficient condition given by Proposition 3.1 and to obtain a good performance in the experimental test. Those facts show the necessity of an adaptive scheme to estimate the mathematical model parameters for every requested operation region and to recalculate the controller gains, doing the application of the optimal nonlinear control to the dehydration process easier.

This section provides details about the development and the implementation of an adaptive scheme, by using inverse optimality approach and identification techniques, which solve the problem to modify the positive scalars W_i, (with $i = 0, 1, 2$) for different operation points.

Systems (2) and (3) are discretized by the Euler forward method [10] as follows:

$$x_{k+1} = \alpha_0 x_k + \alpha_1 x_{k-\bar{h}} + \beta u_k + \bar{\gamma}_0 x_k^2 + \bar{\gamma}_1 x_{k-\bar{h}}^2 + \bar{\gamma}_2 x_k^3 + \bar{\gamma}_3 x_{k-\bar{h}}^3, \tag{13}$$

where coefficients are $\alpha_0 = 1 + T_s a_0$, $\alpha_1 = T_s a_1$, $\beta = T_s b$, $\bar{\gamma}_0 = T_s \gamma_0$, $\bar{\gamma}_1 = T_s \gamma_1$, $\bar{\gamma}_2 = T_s \gamma_2$, and $\bar{\gamma}_3 = T_s \gamma_3$, with k defining the sampling instant time and T_s is the sampling period. Furthermore, the equivalent instant to the delay is defined by $\bar{h} = \frac{h}{T_s}$. With these equivalences, it is possible to obtain an estimate of the parameters of the continuous model given by (2), verifying that the linear part of the system is stable, and then to calculate the optimal control with the estimated plant parameters if the sufficient condition given by Proposition 3.1 is verified. The following procedure describes the proposed algorithm of the adaptive control scheme:

Algorithm 1. *Adaptive optimal nonlinear control*

1) Parameter identification of the discrete model (13) by means of the least square recursive method [9], in which it is possible to take into account the state delay h induced by the recycling pipe. In order to excite the modes of the system, a noised electrical signal represented by a unit step function (the noise added to the signal guarantees the signal persistence) is applied to the actuator; therefore, the step response of the plant is plotted, it means, the required input/output measured vectors for the recursive method are obtained. Then, the regression vector containing the input/output measured variables is given by

$$\bar{x}^T = \left[\; x(\bar{k}),\;\; x(\bar{k}-\bar{h}),\;\; u(\bar{k}),\;\; x^2(\bar{k}),\;\; x^2(\bar{k}-\bar{h}),\;\; x^3(\bar{k}),\;\; x^3(\bar{k}-\bar{h}) \;\right],$$
$$\bar{k} = k - 1,$$

and the vector of the unknown parameters is defined by

$$\bar{\theta}^T = \left[\; \alpha_0,\;\; \alpha_1,\;\; \beta,\;\; \bar{\gamma}_0,\;\; \bar{\gamma}_1,\;\; \bar{\gamma}_2,\;\; \bar{\gamma}_3 \;\right].$$

The initial parameters vector is settled with zero value. Then, when the parameters are obtained for the first time, the estimation error is computed, and this is the initial estimation error, denoted by e_0.

2) Computation of the parameters of the continuous time model (2) as:

$$a_0 = \frac{\alpha_0 - 1}{T_s}, \quad a_1 = \frac{\alpha_1}{T_s}, \quad b = \frac{\beta}{T_s}, \quad \gamma_0 = \frac{\bar{\gamma}_0}{T_s}, \quad \gamma_1 = \frac{\bar{\gamma}_1}{T_s}, \quad \gamma_2 = \frac{\bar{\gamma}_2}{T_s}, \quad \gamma_3 = \frac{\bar{\gamma}_3}{T_s}.$$

3) Verify the stability of the linear part of model (2) (determined by the parameters a_0 and a_1) by the D-partitions method [13] in the continuous domain.

4) The gains of the controller (the positive numbers W_i, $i = 0, 1, 2$) are initialized such that the matrix (10) is definite positive, subject to $W = W_0 + W_1 + hW_2$, and ϵ, $\bar{\alpha}$, $\bar{\beta} > 0$ are given. As the value of W was previously fixed, it is possible to compute the Lyapunov matrix $U(0)$ by using its properties. If the matrix (10) is definite positive,

then the complete type functional $V(x_t)$, given by (5), is a CLKF for system (2) and the optimal control law $u^*(t)$ is calculated to be applied to the plant.

5) The parameters of system (2) are estimated in the next step, and the stability of the linear part is verified again. Additionally, the estimation error is computed, and it is denoted by e_c. If the current estimation error e_c satisfies $e_c < e_0$, and the functional $V(x_t)$ given by (5) is a CLKF for system (2), then the optimal controller $u^*(t)$ is recalculated with the new parameters and applied to the plant; if not, a new parameters identification is computed.
6) In each step, it is verified if the error between the Set Point (SP) and the Process Variable (PV) satisfies the criteria of $\pm 1\%$. If it happens, these are the "best" parameters, and the controller gains are calculated with them for the remain time; if the PV leaves this region, another parameters identification is obtained in order to recalculated the optimal controller.
7) Finally, a stop condition is previously defined: the human user can stop the process at any time or when the product reaches 15% of humidity.

Figure 2 shows a flow diagram of the adaptive algorithm.

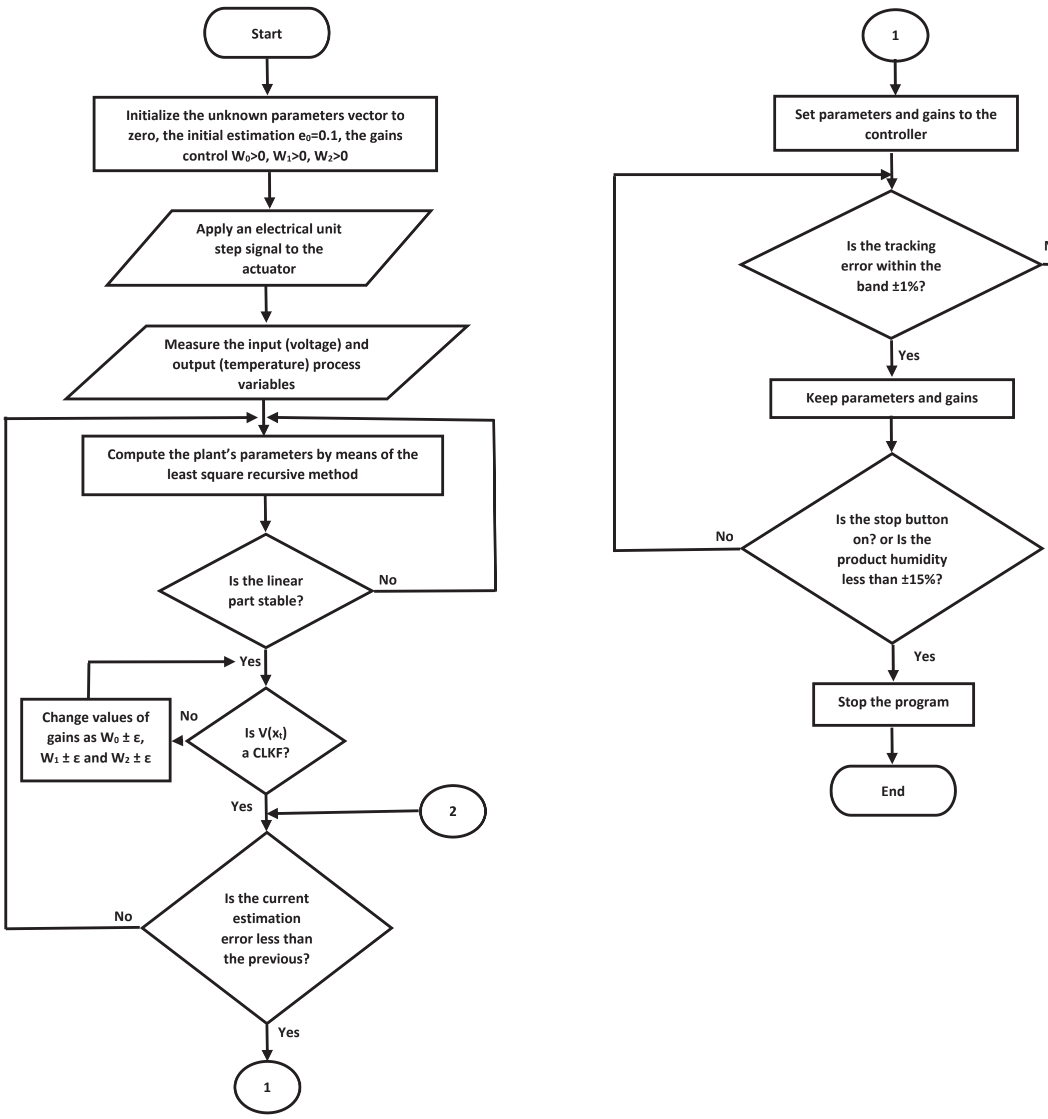

Figure 2. Flow diagram of the adaptive scheme

In the next section the experimental results are presented, and two controllers are considered to regulate the dehydration air temperature, it means, the nonlinear adaptive control and an adaptive Linear Quadratic Regulator by using a linear model.

5. **Experimental Results.** The bananas (Musa balbisiana) were sliced in portions each one with weight of 5 mg approximately; those bananas were chosen with more or less same degree of ripeness, same size and weight. The slices thickness was 5 mm [14] and its diameter was 3 cm. According with the specialized literature [7] three temperatures for the dehydration air was chosen: 45°C, 50°C and 60°C. In order to compare the nonlinear adaptive control performance applied to controlling the air dehydration temperature, an adaptive linear optimal control is programmed. This optimal control is recalculated according with an online parameters identification with fixed pair Q and R of the quadratic performance. In this case, the considered linear model is given by:

$$x_{k+1} = ax(k) + bu(k - \bar{\tau}) + c, \tag{14}$$

where the parameters a, b, and c (Offset) are identified on line and $\bar{\tau}$ is identified off-line by using step response method. For the optimal linear control design, the delay is neglected (the constant time is much greater than the delay); however, for each identified pair a, b, the characteristic equation is calculated and its stability is verified in the discrete domain. With respect to the nonlinear adaptive control, the values W_i, $i = 0, 1, 2$, are settled to 1. The initial parameters for the nonlinear model (13) were adjusted in an arbitrary way, but satisfying the condition (10).

Remark 5.1. *A comparison with an optimal linear control is made, due to the fact that intuitively one can think that the process has a linear behaviour and consequently adaptive linear control is sufficient to regulate it. As it is showed below, it is not true, due to the fact that un-modeled dynamics are latent in the process around some specific operation zone.*

Remark 5.2. *Unlike previous results (please see* [3]*), where the parameters W_i, $i = 0, 1, 2$ have been adjusted every time that the numerical value of the set point changes, in this work, the same numerical values for W_i are used even if the SP is resettled. The reason for this, is that the plant parameters are identified on line. So, the application of the control law given by (12) is better adapted to this SP variation, easier to be implemented and its performance could be improved as it is exposed here below.*

It is important to emphasize that the dryer door is opened every approximately 15 minutes, in order to weigh the product to determine the lost humidity. The indirect humidity measurement is made with the following relation:

$$H = 83\% - \frac{P_i - P_f}{P_i} \times 100,$$

where H is the relative humidity, P_i is the initial weight of the product, P_f is the final weight of the product. The 83% value was obtained from the product with a direct measurement using a humidity sensor $HNZ433A1$. It is clear that the measurement given by this sensor is affected by the environmental humidity, and this humidity present inside of the dryer chamber may affect the humidity measurements of the product. So it is preferred indirect method to weigh the product. According with [14] the objective is to reach 15% of humidity in the product, and then the experiment is finalized when this value is reached in all the slices. Figure 3 shows the loss of humidity in the dried product when the nonlinear and linear adaptive controllers are applied and the set point is set at 45°C.

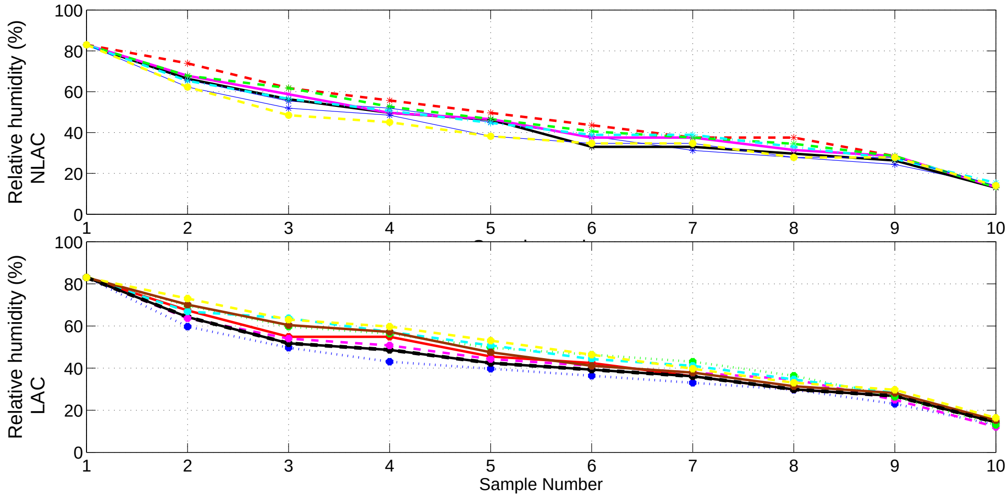

FIGURE 3. (color online) Relative humidity of the sliced banana when the nonlinear adaptive control (NLAC) and linear adaptive control (LAC) are applied (SP adjusted to 45°C)

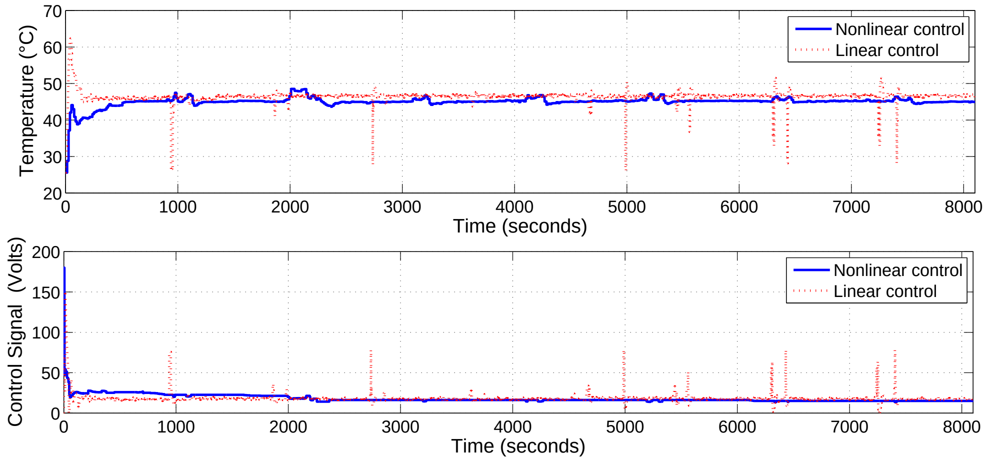

FIGURE 4. Temperature response and control signal with SP adjusted to 45°C using both controllers

More uniform dried products can be observed when the nonlinear adaptive control is used, because less oscillations are present in the temperature, which is shown in the following figures. Figure 4 shows the temperature response and the control signal. The initial condition was 25.58°C.

A better performance is observed when the nonlinear adaptive control is applied, and it is compared to the adaptive optimal linear control strategy. This fact can be explained because a nonlinear model is a better approximation to the real plant than a linear model. Notice that a stationary error is presented when the optimal linear control is used, although the offset is calculated with the estimated parameters and the set point, the estimation error does not allow to reduce this stationary error in the temperature. Figure 5 shows the estimation error for both controllers.

Table 1 shows the last values where the parameters of the nonlinear model converged, and the estimation error is presented, too.

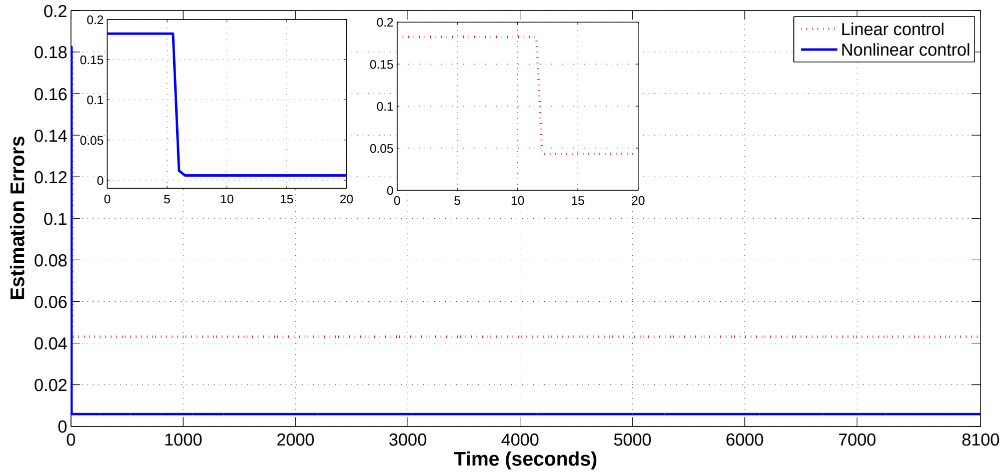

Figure 5. Estimation error for both controllers with SP equal to 45°C

Table 1. Last values for the estimated parameters for the nonlinear model when an SP equal to 45°C is considered

α_0	α_1	β	$\bar{\gamma}_0$	$\bar{\gamma}_1$	$\bar{\gamma}_2$	$\bar{\gamma}_3$	$\widehat{e}$
−1.491172	0.194809	0.002448	1.000382	0.006035	−1.131864	−0.451128	0.005852

(a) Before (b) After

Figure 6. Sliced banana before and after the dehydration process

Furthermore, Figures 6(a) and 6(b) show the sliced banana appearance before and after of the dehydration when the nonlinear adaptive control is applied.

Next, the experimental results when the SP is adjusted to 50°C, are presented. Figure 7 shows the lost humidity by the dried product when both controllers are applied.

Similar drying results are obtained (because the dryer temperature level is higher than that in previous case) when both controllers have been applied; however, as it can be depicted in Figure 8, the performance of the closed loop plant is better when the Nonlinear Adaptive Control (NLAC) is used. The initial condition was at 25.56°C.

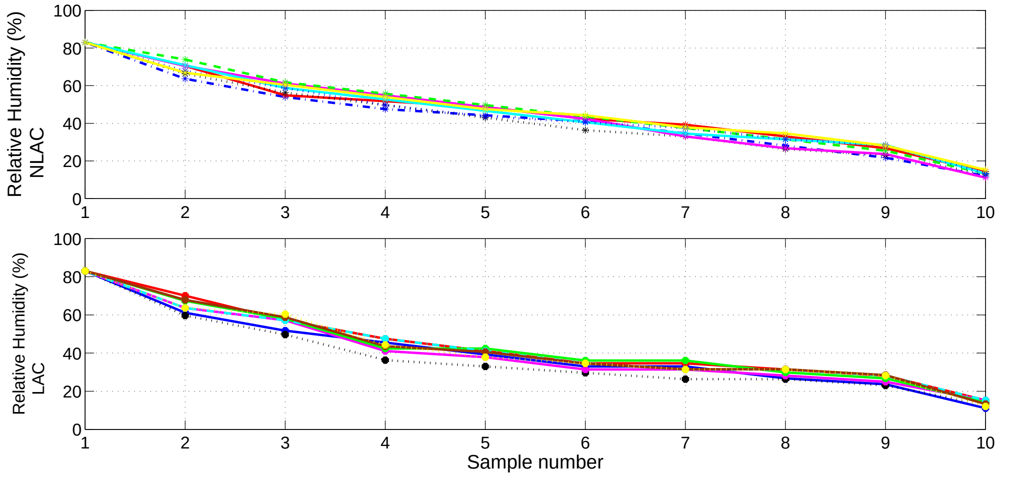

Figure 7. (color online) Relative humidity of the sliced banana when the NLAC and LAC are applied (SP equal to 50°C)

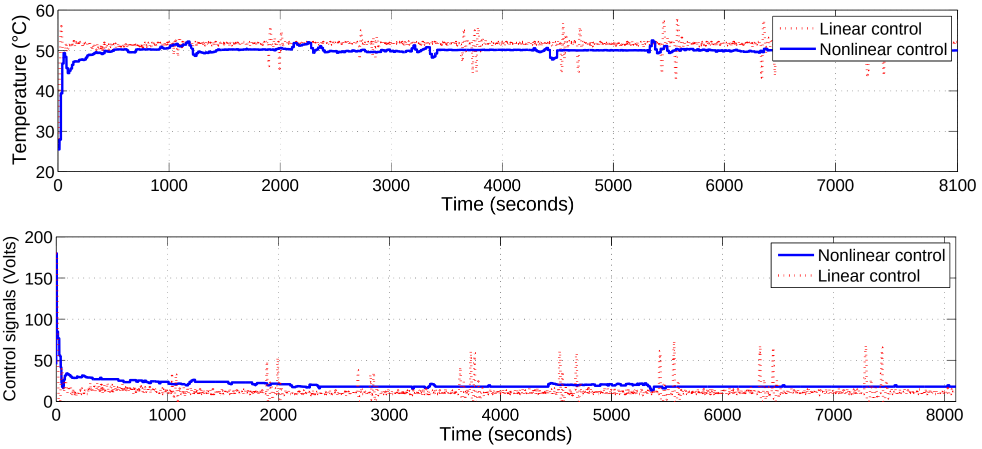

Figure 8. Temperature response and control signal with SP adjusted to 50°C using the nonlinear adaptive control

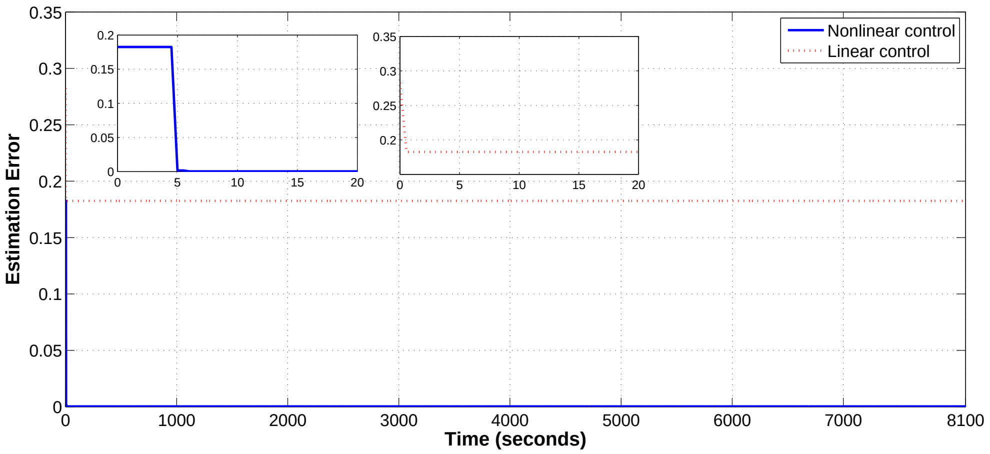

Figure 9. Estimation error with SP equal to 50°C

Again, a better performance is presented by using the nonlinear adaptive control when it is compared with the Linear Adaptive Control (LAC) and as it is shown in Figure 9 the estimation error decreases when the nonlinear model is used.

Table 2 shows the last values where the parameters of the nonlinear model converged and the last calculated estimation error of the process.

Finally, the set point was adjusted to 60°C, and the results are shown below. Figure 10 displays the humidity loss of the dried product when both controllers are applied.

A similar drying performance is obtained, in comparison with the above described experiments. From Figure 11, one can observe that better closed loop performance is

TABLE 2. Last values for the estimated parameters for the nonlinear model when an SP equals 50°C

α_0	α_1	β	$\bar{\gamma}_0$	$\bar{\gamma}_1$	$\bar{\gamma}_2$	$\bar{\gamma}_3$	$\widehat{e}$
−1.380890	0.070492	0.004692	1.005500	0.006911	−1.201833	−0.451582	0.000326

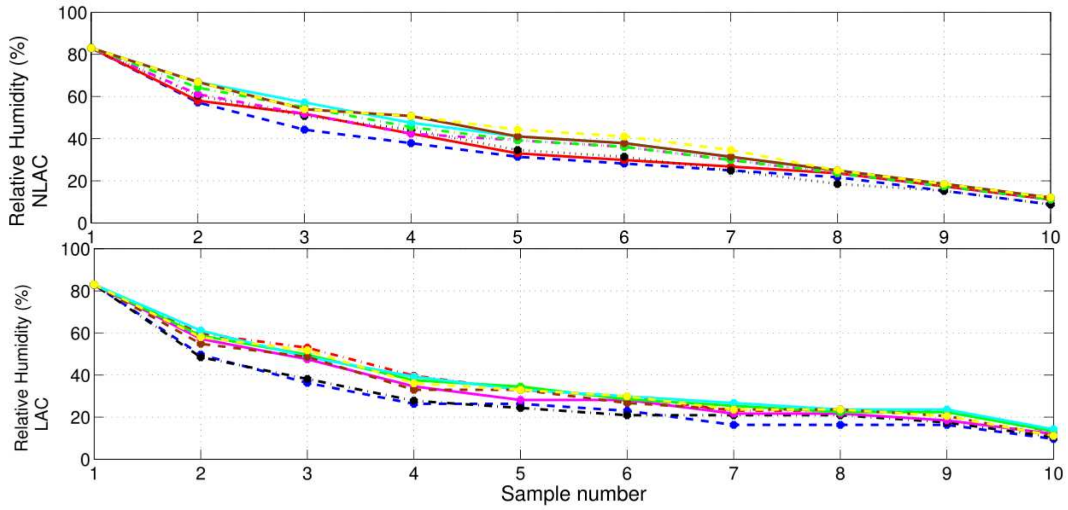

FIGURE 10. (color online) Relative humidity of the sliced banana when the NLAC and LAC are applied (SP equal to 60°C)

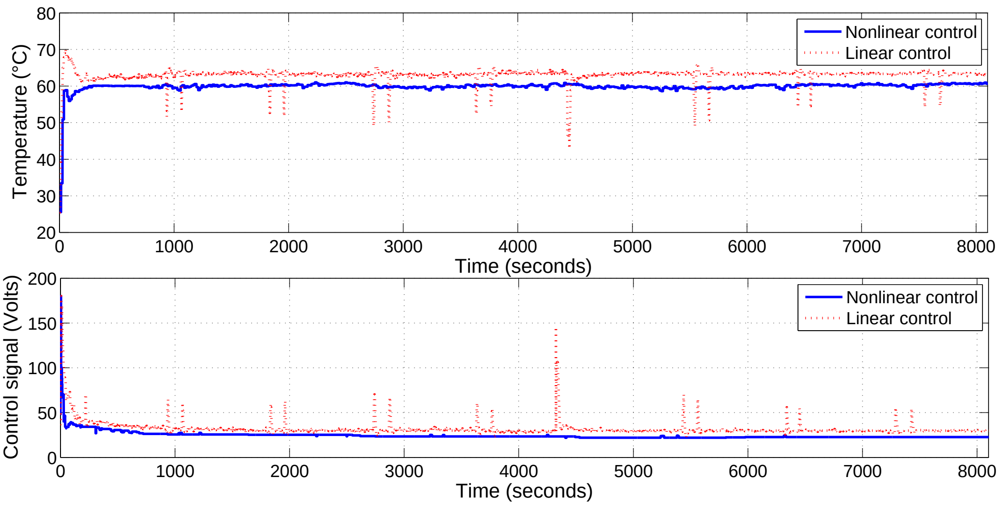

FIGURE 11. Temperature response and control signal with SP adjusted to 60°C using the nonlinear adaptive control

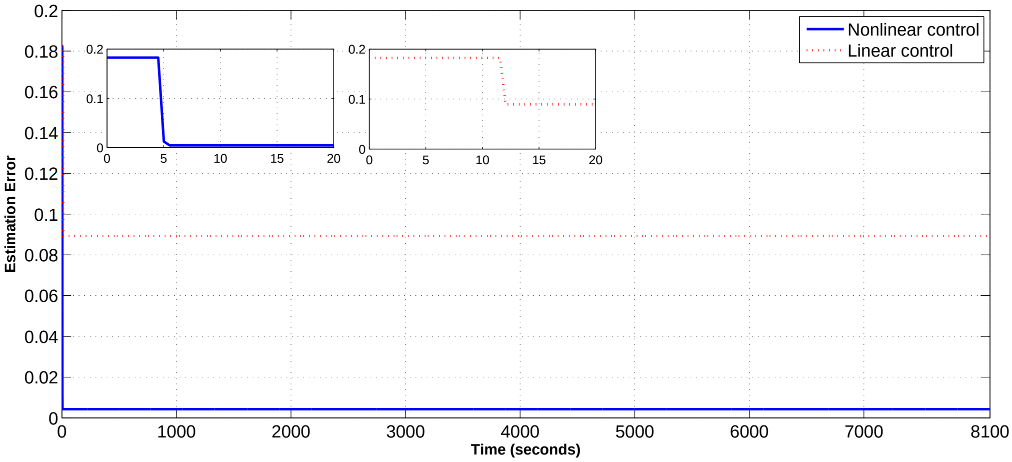

FIGURE 12. Estimation error with SP equal to 60°C

TABLE 3. Last values for the estimated parameters for the nonlinear model when an SP equals 60°C

α_0	α_1	β	$\bar{\gamma}_0$	$\bar{\gamma}_1$	$\bar{\gamma}_2$	$\bar{\gamma}_3$	$\widehat{e}$
−1.215887	−0.184449	0.003926	0.980612	6.909382E-5	−1.340971	−0.452655	0.004238

TABLE 4. Lightness loss of the product using NLAC and LAC

Controller	**Lightness loss (%)**
NLAC 45°C	10.55
NLAC 50°C	8.07
NLAC 60°C	4.6
LAC 45°C	19.46
LAC 50°C	17.9
LAC 60°C	5.3

obtained when the nonlinear adaptive control strategy is applied, with respect to adaptive optimal linear control scheme. In this experiment the initial condition was at 25.50°C.

As Figure 12 shows, once again the estimation error is smaller when the adaptive nonlinear control is applied, in contrast with the closed loop system involving the adaptive optimal linear control.

In Table 3 the final values of both, the parameters of the nonlinear model and the estimation error are shown.

The rates of lightness are measurements by a Hunter Lab colorimeter in the CIE L*a*b parameters, and this color measurement gives the rate of product darkening. Table 4 shows the mean of the lightness loss in the dried product when the parameter L is used to determine it.

In all the considered set points, the nonlinear controller produces a smaller product lightness loss than the linear controller; it is due to the stationary error and the poor performance in the drying temperature air, when the optimal linear control is used.

5.1. **Energy consumption and performance in the control loop.** In this section the performance and the energy consumption of both adaptive controllers (nonlinear and linear strategies) are analyzed. On the one hand, the performance of the closed loop

Table 5. Comparison of energy consumption of the NLAC and LAC

Set Point	**Energy consumption – LAC (Wh)**	**Energy consumption – NLAC (Wh)**
45°C	60.24	57.09
50°C	74.15	70.28
60°C	228.12	98.80

Table 6. Comparative table of the numerical values for the IAE using the NLAC and LAC

Set Point	**IAE-LAC**	**IAE-NLAC**
45°C	12, 776.13	4, 528.62
50°C	13, 709.44	3, 663.73
60°C	25, 356	3, 643.92

system is measured by using the Integral Absolute Error (IAE) criterion. While the energy consumption is computed with the instantaneous power rate by a period time. Table 5 shows the energy consumption by the closed loop plant for each one of controller.

Although both controllers are optimal in different senses, only when the SP is set at 70°C, an important energy saving is observed when the adaptive nonlinear controller is used, instead of the adaptive linear control: 56.6% of energy savings. Moreover, with respect to the performance of the closed loop plant, Table 6 displays the numerical values of the performance index IAE. It is clear that some advantages can be obtained if a nonlinear model and an adaptive optimal control are used to regulate the dehydration air of the process, among which are energy savings, smaller lightness loss and good performance of the closed loop plant.

The advantages of the adaptive approach are listed below.

1) The plant parameters estimation is carried out on line, reducing the required time to the identification of the process in contrast to the off line procedure.
2) The change of the parameter values, due to the initial condition affected and related by the external environment temperature and humidity, is taken into account with the recursive method in every step, and hence the performance of the plant response is improved.
3) The controller gains are initialized with a fixed value; however, during the process, they can be updated in function of the error in the process variable, while in [3], they are set only one time.

6. **Conclusions.** In the present article an adaptive optimal nonlinear control is experimentally tested and acceptable results are obtained. Although some previous results have been recently presented [3, 4], using the inverse optimality approach, they have some difficulties which should be mentioned: some parameters (parameters plant and additional parameters which guarantee the stability of closed loop plant) have to be adjusted for each SP. With the adaptive scheme presented in this work, this problem is solved and another application is also considered in this paper: sliced bananas dehydration process. When the optimal adaptive nonlinear control performance is compared with the performance given by an optimal adaptive linear control, some advantages can be highlighted, which are smaller lightness loss in the product and energy savings in the process. Future works includes nutrients loss analysis for the dried bananas when some specific controllers are applied to regulating the dehydration air temperature.

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